On singularities of the mixed state phase

Rajendra Bhandari

Raman Research Institute,
Bangalore 560 080, India.
email: bhandari@rri.res.in

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In an interesting contribution to the physics of interference of particles with internal degrees of freedom, Sjöqvist et.al. [1] have shown that if a beam of such particles, in a mixed state with a density matrix ρ_0, is split in a Mach-Zhender interferometer and a unitary transformation U_i acts on the space of N internal states in one of the two paths, a phase difference given by arg $\mathrm{Tr}(U_i\rho_0)$ is introduced between the two beams. Here "phase difference" is defined as the shift in the maximum of the interference pattern. For pure states $\mid \psi_0 >$ this quantity reduces to the phase shift $\arg(< \psi_0 | \, U_i \, |\psi_0 >)$ defined by Pancharatnam [2]. The purpose of this note is to point out that the definition of the mixed state phase as in ref.[1] breaks down at points in the parameter space where $\mid Tr(U_i\rho_0) \mid$=0. Eqn.(8) in the paper shows that at such points the interference pattern has zero contrast. Such points constitute singularities of phase involving discontinuous phase jumps. In the pure state case, such singularities have been demonstrated theoretically and experimentally for N=2 using the two polarization states of light [3, 4, 5, 6]. The mixed state phase involves singularities in new kinds of parameter spaces which include variables representing decoherence of quantum states. We show that the singularities make interpretation of even simple interference experiments like the ones analyzed in ref. [1], in terms of the proposed phase, nontrivial.

Consider a beam of spin-1/2 particles in a mixed state with a density matrix $\rho_0 = \mathrm{diag}[(1+r)/2, (1-r)/2]$ in the basis of $\mid z\pm >$; $\mid z\pm >$ being the eigenstates of z-component of the spin and r the degree of polarization. The beam is split symmetrically in an interferometer and, to keep things simple, a unitary transformation U_i, diagonal in the $\mid z\pm >$ basis, is applied in one of the two paths by means of a variable magnetic field along $\hat{z}$. This results in the $| z\pm >$ states acquiring a phase factor $e^{\pm i\delta}$. The mixed state phase ϕ is given by $Tr(U_i\rho_0) = [cos\delta + i\ r\ sin\delta] = c\ e^{i\phi}$. In the plane of the parameters (r, δ), phase singularities occur at the points $(0, (j + 1/2)\pi)$, where c vanishes; j being any integer. The interference pattern for the unpolarized case ($r = 0$) can be looked upon as a superposition of two cosine intensity patterns with equal amplitude for the two spin eigenstates, moving in the opposite directions as δ is varied [5]. When $\delta = (j + 1/2)\pi$, the relative phase shift of the two patterns equals $(2j + 1)\pi$ and the superposition yields uniform illumination, making the phase indeterminate. A counterclockwise (clockwise) circuit in the (r, δ) plane around any one of these singularities yields a value 2π (-2π) for the total phase change $\int d\phi$.

Fig.1 shows a set of typical phase shifts acquired by a mixed state as a function of δ. In each case the phase shift equals π in magnitude for a variation of δ through π, is linear for pure states i.e. for r=$\pm$1 (curves A and B), is highly nonlinear near the singularity at (r=0, $\delta = \pi/2$) (curves C and D), and changes sign with the sign of r ($d\phi/d\delta$ has the same sign as r). For r=0, the phase shift is indeterminate, contrary to the claim in ref.[1] of its being equal to π. The neutron interferometer experiments cited therein, done with unpolarized neutrons (r=0), do not measure the *phase shifts* as shown in fig.1. These can however be measured in an interference experiment with neutrons or with polarized light if r can be varied and phase shifts measured in the experiment. The basic topological effect namely a $2n\pi$ phase change (n being integer) resulting from a circuit around one or more singularities can in fact be measured without a close approach to the singularities, making experiments easier. In an experiment with spin>1/2, even a circuit around a single singularity could result in a $2n\pi$ phase shift with $n > 1$, yielding information about the spin quantum number. A modulo 2π description of the phase, as in ref.[1] and in early work on the geometric phase, thus misses an essential aspect of the physics in the problem.

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Figure Caption:

FIG.1: Computed phase shift $\int d\phi$ as a function of applied magnetic field for (A) r=1, (B) r=-1, (C) r=.001 and (D) r=-.001.